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LETTER TO THE EDITOR

Dynamics of fluxon lattice in two coupled Josephson junctions

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Abstract. We study theoretically the dynamics of a fluxon lattice (FL) in two coupled Josephson junctions. We show that when the velocity of the moving FL exceeds certain values ($V_{a,b}$), sharp resonances arise in the system which are related to the excitation of the optical and acoustic collective modes. In the interval (V_a, V_b) a reconstruction of the FL occurs. We also establish that one can excite localized nonlinear distortions (dislocations) which may propagate through the FL and carry an arbitrary magnetic flux.

In recent years many papers have addressed the dynamics of the fluxon lattice in layered superconductors and, in particular, in high T_c superconductors. In the absence of a magnetic field, the spectrum of collective oscillations in the long wave limit has a threshold frequency which coincides with the Josephson frequency ω_J [1–7]. If a magnetic field is applied, the vortex lattice is formed in a superconductor, and the spectrum of collective modes is changed. In particular, an acoustic-like mode related to the vortex lattice oscillations arises. The spectrum of the fluxon lattice (FL) arising in a parallel magnetic field was calculated in [6–8].

The collective modes may be excited by an external ac field and also by a dc current (j_{dc}) across the layers. In the latter case, the collective modes are excited if the velocity of the moving FL coincides with the phase velocity of the collective modes. This effect has been studied in detail for the case of a long Josephson tunnel junction [9] and was studied in a theoretical paper [10] recently for the case of layered superconductors.

In this letter we consider two coupled, long Josephson junctions and study the excitations of small amplitude collective modes as well as the excitations of nonlinear perturbations (dislocations) of the FL in such a system by a dc current. The comparative simplicity of equations governing the FL dynamics in this system allows one to analyse effects arising in this system in detail, and to understand the behaviour of more complicated structures such as layered superconductors. It will be shown, in particular, that nonlinear excitations (dislocations) may arise in the system due to the dc current, and that these excitations can carry an arbitrary magnetic flux (larger or smaller than the magnetic flux quantum Φ_0). We note that the system under consideration was analysed in the absence of a magnetic field in [11–13].

Let us consider the system shown in figure 1. We assume that different currents may be passed through junctions 1 and 2, i.e. a current through the middle superconductor can be driven independently from currents through the outer superconducting electrodes. For simplicity we assume that the junctions are identical, i.e. they have equal critical currents,

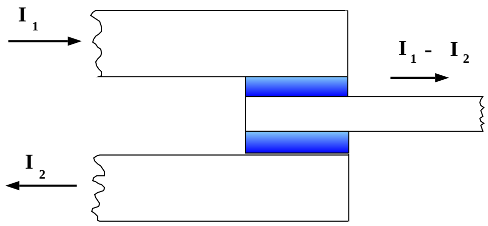

Figure 1. A schematic diagram of the structure under consideration.

etc. Equations describing the dynamics of the coupled Josephson junctions have been obtained in a number of works [4–8, 11–13]. The magnitudes of the magnetic field in junctions 1 and 2 are related to the phase difference $\varphi_{1,2}$ through the expression

$$H_{1,2} = (1/2)\partial_x\big[\varphi_{1,2} + \gamma\varphi_{2,1}\big]. \tag{1}$$

Here $H_{1,2}$ are the dimensionless magnitudes of the magnetic field in junctions 1 and 2. We choose the quantity $H_0 = \Phi_0/2\pi\lambda^2$ as a unit of measurement of the magnetic field and the London penetration depth as a length unit. Φ_0 is the magnetic flux quantum, $\varphi_{1,2}$ is the phase difference in junctions 1 and 2 and $\gamma = \exp[-2d]$, where $2d$ is the thickness of the middle electrode (in units of λ). We assume that the characteristic scale of spatial variations in $\varphi_{1,2}$ is much greater than unity (i.e. than λ), and also that the thickness of the outer superconducting layers is greater than λ.

The current through the junctions can be expressed through the corresponding component of $(\nabla \times H)$ and can be related to the quasiparticle and Josephson currents. We obtain

$$2l_J^2\partial_x H_{1,2} = \left(\partial_{tt}^2 + \alpha\partial_t\right)\varphi_{1,2} + \sin\big(\varphi_{1,2}\big) - \eta_{1,2}. \tag{2}$$

Here $l_J = (cH_0/8\pi j_c\lambda)^{1/2}$ is the dimensionless Josephson penetration length, j_c is the critical Josephson current density, $\alpha = \hbar/2e\rho_{qp}j_ct_0$ is the damping constant, ρ_{qp} is the junction resistivity due to quasiparticle tunnelling. Time is measured in units $t_0 = \sqrt{\hbar C/2ej_c}$, where C is the junction capacitance (per unit area). The constants $\eta_{1,2}$ are dimensionless currents (in units of j_c) through junctions 1 and 2. It can easily be shown that the magnetic flux in the system equals

$$\Phi = \int_0^L \mathrm{d}x\,\partial_x\big(\varphi_1 + \varphi_2\big). \tag{3}$$

Substituting for $H_{1,2}$ in (2) from (1), we obtain a set of two coupled equations for $\varphi_{1,2}$

$$l_J^2\partial_{xx}^2\big[\varphi_{1,2} + \gamma\varphi_{2,1}\big] = \big(\partial_{tt} + \alpha\partial_t\big)\varphi_{1,2} + \sin\big(\varphi_{1,2}\big) - \eta_{1,2}. \tag{4}$$

We now introduce the new functions $\varphi_\pm = (1/2)(\varphi_1 \pm \varphi_2)$. Summing and subtracting equation (4), we obtain the new equation for $\varphi_\pm$

$$l_\pm^2\partial_{xx}^2\varphi_\pm = \big(\partial_{tt} + \alpha\partial_t\big)\varphi_\pm + \sin\big(\varphi_\pm\big)\cos\big(\varphi_\mp\big) - \eta_\pm. \tag{5}$$

Here $l_\pm^2 = l_J^2(1 \pm \gamma)$ and $\eta_\pm = (\eta_1 \pm \eta_2)/2$. Equations (5) describe the dynamics of two coupled Josephson junctions. We use them for studying the FL. Let us assume that a magnetic field parallel to the planes of the Josephson junctions is applied and a dense FL arises in junctions 1 and 2. In the stationary state (and sufficiently high magnetic fields) the solution for $\varphi_{1,2}$ can be easily found from equation (4): $\varphi^{(s)}_{1,2} = \mathcal{H}x \pm (\pi/2) + \psi^{(s)}_{1,2}$, where $\psi^{(s)}_{1,2} \approx \mp(l_-\mathcal{H})^{-2}\cos(\mathcal{H}x)$. This solution is valid provided

$$l_-^{-1} \ll \mathcal{H} \ll 2\pi. \tag{6}$$

Then the right-hand side of this condition means that equation (1) is valid, i.e. the characteristic scale of $\varphi_{1,2}$ variation along the x-axis is greater than the London penetration depth. The left-hand side of this condition means that the FL is dense, and spatial oscillations of $\varphi_{1,2}$ are small. The field in the junctions $\mathcal{H}$ is related to the external field by $\mathcal{H} = 2H_e/(1+\gamma)$. The expression for $\varphi^{(s)}_{1,2}$ given above describes two fluxon chains each of which is shifted by a half-period with respect to each other.

Let us now consider solutions describing the motion of the FL driven by the dc currents $\eta_{1,2}$ (the currents $\eta_{1,2}$ may differ from each other). We seek the solution of equation (5) in the form of a travelling wave, assuming that the junctions are long enough and neglecting reflected waves

$$\varphi_{1,2} = \mathcal{H}x - Vt + \psi_{1,2} + \theta_{1,2} + \theta^{(0)}_{1,2} \tag{7}$$

where $\psi_{1,2}$ is the rapidly oscillating part of $\varphi_{1,2}$ in space with a period of $2\pi/\mathcal{H}$. $\theta_{1,2}$ is the slowly varying part and $\theta^{(0)}_{1,2} = 0$ (for 1) and π (for 2). A similar representation was used in [14–16] for finding the shape of the supersolitons (dislocations) in the FL and in [8] for finding the spectrum of acoustic-like oscillations of the FL. For the rapidly oscillating part $\psi_\pm = (\psi_1 + \psi_2)/2$, we have from equation (5)

$$\begin{aligned}
\psi_+ &= -\frac{\sin(\theta_-)}{|D_+|^2}\left\{b_+\cos\big(Y+\theta_+\big) + \alpha V\sin\big(Y+\theta_+\big)\right\}\\
\psi_- &= \frac{\cos(\theta_-)}{|D_-|^2}\left\{-b_-\sin\big(Y+\theta_+\big) + \alpha V\cos\big(Y+\theta_+\big)\right\}
\end{aligned} \tag{8}$$

where $Y = \mathcal{H}x - Vt$ and $\theta_\pm = (\theta_1 \pm \theta_2)/2$, $b_\pm = V_\pm^2 - V^2$, $D_\pm = b_\pm + \mathrm{i}\alpha V$ and $V_\pm = l_\pm\mathcal{H}$. Assuming that $\psi_\pm$ are small (i.e. $|\psi_\pm| \ll 1$), and expanding $\sin(\varphi_\pm)$ in powers of $\psi_\pm$, we obtain from equations (5) and (8) the equations for the slowly varying part $\theta_\pm$ in the main approximation

$$\begin{aligned}
l_+^2\partial^2_{xx}\theta_+ &= \Big(\partial^2_{tt} + \alpha\partial_t\Big)\theta_+ + \alpha VA_-\cos\big(2\theta_-\big) + \alpha VA_+ + \alpha V - \eta_+\\
l_-^2\partial^2_{xx}\theta_- &= \Big(\partial^2_{tt}\alpha\partial_t\Big)\theta_- + B\sin\big(2\theta_-\big) - \eta_-.
\end{aligned} \tag{9}$$

Here $4A_\pm = |D_-|^2 \pm |D_+|^{-2}$ and $4B = b_-|D_-|^{-2} - b_+|D_+|^{-2}$. Equation (9) describes the dynamics of the FL. The phase θ_+ is a local displacement of the FL as a whole and the phase θ_- determines a relative displacement of two fluxon chains.

Consider the stationary case when the currents $\eta_\pm$ are absent and the FL is motionless ($V = 0$). Linearizing equation (9), we obtain for the spectrum of the collective modes (for simplicity we neglect the damping)

$$\omega^2 = \kappa^2 l_+^2 \qquad \omega^2 = \omega_0^2 + \kappa^2 l_-^2. \tag{10}$$

Here $\omega_0^2 = \gamma/[(1-\gamma^2)l_J^2\mathcal{H}^2]$ is the threshold frequency for the optical branch. It decreases with increasing $\mathcal{H}$. The first expression in equation (10) describes the acoustic branch of the FL oscillations. Similar modes exist in layered superconductors [6–8]. These modes are independent of each other. In the acoustic (optical) mode the phase θ_+ (θ_-) is perturbed. Consider now excitations of these modes by the moving FL.

Let us assume that a current $\eta_1 = \eta_2$ flows through both junctions (then $\eta_+ = n_2 = \eta$ and $\eta_- = 0$). Then the stable solution of equation (9) is: $2\theta_- = 0$ for $V < V_a$, $V > V_b$

and $2\theta_- = \pi$ for $V_a < V < V_b$. Here $V_{a,b}$ are the roots of the equation $B(V) = 0$. The solution $\theta_- = 0$ (π) corresponds to a positive (negative) value of B. For the case of a small damping α (i.e. $\alpha \ll V_\pm$) we have $V_{a,b}^2 \cong V_\pm^2[1 \mp \alpha^2/(V_+^2 - V_-^2)]$.

If the velocity of the moving FL exceeds ($V_a/\mathcal{H}$), a reconstruction of the moving FL occurs (see figure 2). If the velocity of the FL increases further and exceeds ($V_b/\mathcal{H}$), the initial triangular form of the FL is restored.

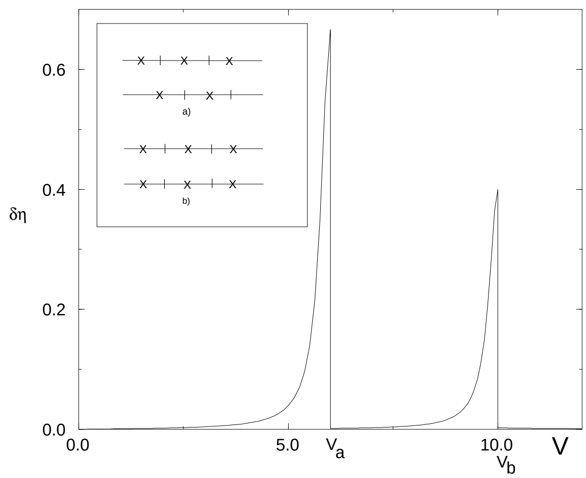

Figure 2. The deviation of the current from Ohm's law ($\delta\eta = \eta - \alpha V$)) due to excitation of the collective modes versus voltage (we used $\alpha = 0.5$). The positions of fluxons (crosses) in both junctions are shown inset for $V > V_a$, $V_b < V$ (a) and $V_a < V < V_b$ (b).

The form of the current–voltage characteristics $\eta(V)$ may be easily found from equation (9) averaged in space. We obtain

$$\eta - \alpha V = 2\alpha V \begin{cases} |D_-|^{-2} & V < V_a,\ V > V_b \\ |D_+|^{-2} & V_a < V < V_b. \end{cases} \tag{11}$$

There are two peaks in the $\eta(V)$ characteristics, one of them corresponds to the velocity of the optical mode (at large κ) and the second corresponds to the velocity of the acoustic mode. In addition there are two jumps at voltage $V = V_a$ and $V = V_b$ corresponding to the FL reconstruction. If the damping constant α is small, then $V_{a,b} \approx V_\pm$. In figure 2 we present the form of the $\eta(V)$ curve assuming for simplicity that α does not depend on V.

Let us assume that a bias current flows in the middle superconducting electrode ($\eta_- \neq 0$). One can see that the second equation of (9) is similar to an equation describing a single Josephson junction. This equation describes nonlinear distortions in the FL. These distortions (dislocations) were analysed in [14] where it was shown that they arise as kinks in the FL in two, slightly different coupled Josephson junctions. Similar distortions (supersolitons) may arise in a single long Josephson junction whose parameters are modulated in space [15]. The characteristic length of dislocations l_-/B depends on both the magnetic field and the applied voltage (or current). Dislocations are created at $\eta_- > B(V, \mathcal{H})$ and can propagate through the system in an interval of η_- above and below B (Fiske steps [9]). If the damping is small, the dislocation has the well known fluxon form $2\theta_- = 4tg^{-1}\exp(\xi)$, where $\xi = (x - ut)/l_d$ and $l_d = l_-/\sqrt{B}$. The velocity u is related to η_- via the well known formula [16], which in the slow velocity limit is reduced to

$u = (\pi/4)\eta_- l_d/\alpha$. According to equation (9), a dislocation arising in a moving FL causes a perturbation of the phase θ_+. Substituting the expression for θ_- into (9), we obtain for θ_+

$$\partial^2_{\xi\xi}\theta_+ + \beta\partial_\xi\theta_+ = -s\cosh^{-2}\xi \tag{12}$$

where $\beta = \alpha u l_d/[l_+^2 - u^2]$ and $s = 2\alpha V A_- l_d^2/[l_+^2 - u^2]$. If the FL velocity V lies in the intervals $(0,\ V_a)$, $(V_b,\ \infty)$, we obtain from equation (9) $\eta_+ = \alpha V(A_+ + A_-)$.

From equation (12) we obtain

$$\partial_\xi\theta_+ = -s\int_{-\infty}^{\xi} \mathrm{d}\xi_1 \cosh^{-2}\xi_1 \mathrm{e}^{\beta(\xi_1-\xi)}. \tag{13}$$

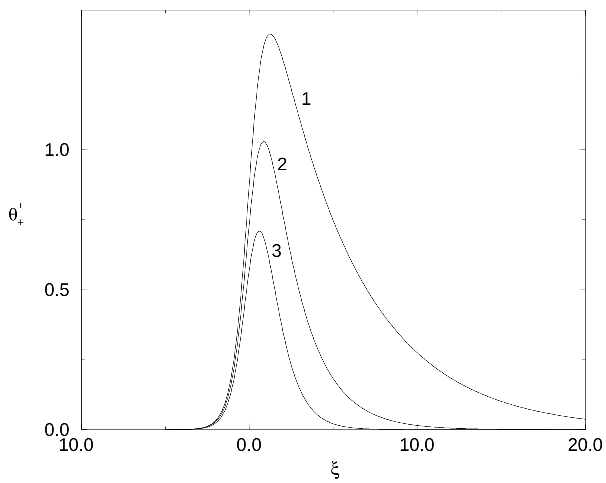

Figure 3. The spatial dependence of the magnetic field ($\theta'_+ = \partial_\xi\theta_+$) in the dislocation for different values of β: $\beta = 0.2$ (1), 0.2 (2), 1.0 (3).

The spatial dependence of the dislocation $\theta_+(\xi)$ is shown in figure 3 for different values of β. The expression for $\theta_+(\xi)$ (13) is valid provided that the characteristic size of the dislocation β^{-1} is less than the junction length L. Let us calculate the magnetic flux carried by a dislocation in a moving FL. Substituting expression (13) into equation (3), we have for magnetic flux in the system

$$\Phi = 2(\mathcal{H}L) + 2\int \mathrm{d}x\,\partial_x\theta_+. \tag{14}$$

The first term in equation (14) is the magnetic flux in the system in the absence of a dislocation. The second term is the magnetic flux Φ_d carried by a dislocation. With the help of equation (13) we obtain for Φ_d

$$\Phi_d = -\frac{8VA_-}{u(V)} l_d. \tag{15}$$

One can see that the flux Φ_d depends both on the velocity of the dislocation and the velocity of the FL as a whole, turning to zero at $V = 0$. Therefore, a dislocation in the FL is a localized distortion which can move under the action of an external force (difference of the currents η_-) and carry an arbitrary magnetic flux, the magnitude of which is determined by currents η_+ and η_-.

In summary we have analysed the dynamics of a dense FL in a system of two coupled Josephson junctions. Acoustic and optical collective modes may propagate in the FL. If the

FL is moving in the presence of a dc current, a resonance excitation of the modes takes place when the FL velocity coincides with the limiting velocity V_- of the optical mode, or with the velocity of the acoustic mode V_+. In the interval (V_-, V_+) the FL is reconstructed. It was also shown that if the currents η_1 and η_2 through the junctions are different, localized distortions (dislocations) may be created in the FL. They carry an arbitrary magnetic flux and may lead to a non-Josephson generation.

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